Journal of Applied Developmental Psychology 35 (2014) 444–454

Contents lists available at ScienceDirect

Journal of Applied Developmental Psychology

Early use of decomposition for addition and its relation to base-10 knowledge ☆

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ARTICLE INFO

Article history:
Received 6 November 2013
Received in revised form 24 July 2014
Accepted 24 July 2014
Available online 23 October 2014

Keywords:
Addition strategy
Base-10
Decomposition
Mathematics
Cross-national
Early childhood

ABSTRACT

The early use of decomposition for addition has been linked to future mathematics achievement. The present study examined kindergartners' performance on addition problems, focusing on their use of the decomposition strategy and the factors related to the frequency with which they chose it. Single- and multi-digit addition problems were presented to kindergartners from US, Russia and Taiwan ($N = 182$). As expected, kindergartners used a variety of strategies to solve the problems. They were more likely to use decomposition on complex problems involving carryover or multi-digit operations. Critically, their use of base-10 decomposition was related to their knowledge of base-10 number structure. These relations were similar across all three nations. Implications for understanding mathematical development and designing early mathematics instruction are discussed.

A key goal of early mathematics instruction is to develop students' ability to efficiently and accurately perform basic arithmetic operations (National Council of Teachers of Mathematics [NCTM], 2000, 2006). Both researchers and educators have emphasized the importance of acquiring these skills in a meaningful way — in connection to a conceptual understanding of numbers and numeric relations, rather than mechanically learning a set of procedures. Thus, researchers have increasingly focused their investigation on the strategies children use when solving arithmetic problems and how children select among the different strategies available to them (Canobi, Reeve, & Pattison, 2003; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Geary, Bow-Thomas, Liu, & Siegler, 1996; Lindberg, Linkersdörfer, Lehmann, Hasselhorn, & Lonnemann, 2013).

Examining children's strategy choices provides insight into their understanding of the numeric structure and relations among numbers. For example, when children decompose the addends in an addition problem in order to simplify the computation, they demonstrate an understanding of the composition of numbers (e.g., $7 + 4 = 7 + 3 + 1 = 10 + 1 = 11$). Several studies have found a relation between the frequency with which children use more advanced and efficient mental computational strategies and mathematics performance. In particular, starting in first grade, use of a decomposition strategy has been associated with better performance on a variety of tasks that involve computation (Carr, Steiner, Kyser, & Biddlecomb, 2008; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Geary, Hoard, Nugent, & Bailey, 2013). For example, Fennema, Carpenter, Jacobs, Franke & Levi (1998) found that first graders who used the mental decomposition strategy demonstrated deeper conceptual understanding of addition and subtraction on transfer problems than those who did not rely on this strategy.

Thus, it is important to better understand early individual differences in the use of decomposition and the factors that contribute to those differences. In the present study, we examined the frequency with which kindergarten students used a base-10 decomposition strategy to solve addition problems. We were particularly interested in understanding whether and how these differences in strategy choice were related to problem characteristics and children's understanding of the base-10 numeric structure.

Addition strategies

Addition problems can be solved using a variety of strategies. Generally, children use three types of addition strategies: counting, decomposition, and retrieval (Geary, Bow-Thomas, Liu & Siegler, 1996; Geary, Fan, & Bow-Thomas, 1992; Shrager & Siegler, 1998). *Counting* is

☆ Thanks are due to the children, teachers, and principals who participated in this research; to Weng-Feng Lai, and Amy Hachigian for assistance coding the data, and to Boston College's Provost Office who supported this research.

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http://dx.doi.org/10.1016/j.appdev.2014.07.002
0193-3973/

among the first to emerge in children's repertoire of addition strategies. To begin with, counting strategies involve enumerating both of the addends ("*count-all*"). This strategy has been observed in children as young as 3-year-olds. During the preschool years, children acquire a more advanced counting strategy, which involves counting up from one addend the value of the second addend ("*count-on*"; Siegler & Robinson, 1982). By the age of five, most children have mastered counting strategies. Through experience using these strategies, children begin to commit simple addition facts to memory, which leads to the emergence of memory-based strategies, such as retrieval and decomposition. The *retrieval* strategy involves recalling the solution to a problem as a number fact stored in memory, rather than active computation. *Decomposition* involves transforming the original problem into two or more simpler problems, using either a previously memorized number fact ("*fact-based decomposition*") or the base-10 properties of the number system ("*base-10 decomposition*"). An example of the latter is solving $6 + 5$ by adding 6 and 4 to get to 10, and then 1 more; an example of the former is using a memorized fact $6 + 6 = 12$, and then subtracting 1 to solve $6 + 5$. Children's skill in executing these strategies continues to improve over the elementary school years, gradually being extended to more complex problems.

Children are generally able to use more than one strategy to solve arithmetic problems (Ashcraft, 1982; Carpenter & Moser, 1984; Geary, 1994; Siegler & Shrager, 1984). For example, when asked to solve a series of problems in one session, a child might count-on to solve one problem, retrieve the answer from memory to answer the next problem, and use decomposition to solve another problem. Yet, among children of the same age there are individual differences in terms of the frequency with which they use different strategies. Further, with development, the extent to which children rely on particular strategies changes such that the frequency of counting strategies decreases and the use of retrieval and decomposition increases (e.g., Carpenter & Moser, 1984; Fuson, 1992; Geary, Hoard, Byrd-Craven & DeSoto, 2004; Siegler & Shrager, 1984).

Strategy choice

Children's adaptive selection of strategies is a key process underlying development in mathematics as well as other domains (Siegler, 1996). The Overlapping Waves Theory of development posits that (a) at any one time, children know and are able to use multiple strategies to solve problems in a given domain; (b) these strategies compete with each other over prolonged periods of time; and (c) cognitive development and improved performance in a domain involves changes in the relative frequency of use of these strategies, with new strategies sometimes being added and others sometimes ceasing to be used.

This theory describes strategy choice as a competition between accuracy and efficiency (Shrager & Siegler, 1998). Early on children's predominant strategy may be a relatively inefficient one, such as count-all, that leads to a correct response. As children learn new strategies and are able to execute them correctly, their predominant approaches change toward more efficient strategies. At any given time, strategy choice is constrained by characteristics of the problem being solved, such as problem difficulty, and of the individual solving the problem, such as prior knowledge (Geary, Hoard, Byrd-Craven & DeSoto, 2004; Kerkman & Siegler, 1993; Laski et al., 2013; Lemaire & Callies, 2009; Siegler, 1988).

Problem characteristics

The frequency with which children use a particular strategy is related to problem difficulty. Children tend to use retrieval on easier problems for which it is likely to lead to an accurate response, but select an alternative strategy that involves computation on more difficult problems for which retrieval is less likely to lead to an accurate response (Lemaire & Callies, 2009; Siegler, 1996).

The likelihood of choosing a specific computational strategy, thus, varies with problem characteristics. Evidence suggests that children are unlikely to use the most advanced computational strategy available to them unless the difficulty of the problem demands it. Increasing problem difficulty promotes the use of more advanced computational strategies, in order to maximize efficiency while still maintaining accuracy. For instance, Siegler & Jenkins (1989) investigated the use of counting strategies in preschoolers. They found that the frequency with which children used the more advanced count-on strategy (as opposed to count-all) was higher when children were presented with complex problems involving an addend above 20 than when they were presented with simple problems involving addends between 1 and 5.

Given the findings about what influences preschoolers' choice of counting strategies, it is possible that older children, having a better understanding of numeric structure, would be more likely to use a decomposition strategy on problems for which counting is relatively inefficient (e.g., $26 + 8$). In fact, a cross-national study by Geary, Bow-Thomas, Liu & Siegler (1996) showed that Chinese kindergartners used decomposition more frequently on problems with sums greater than 10 than on problems with sums less than 11. At the same time, their American peers did not show a similar pattern as a function of problem difficulty. It should be noted, however, that children in this study were given addition problems involving only single-digit addends. In the present investigation, we extended the range of problems to include mixed-digit addition, where one of the addends is a single-digit and the other is a double-digit number, to explore whether these problems push kindergartners to use a decomposition strategy more often than in previous studies.

Students' prior knowledge

While problem difficulty can affect the frequency with which children use different strategies, children's strategy choice is also constrained by individual differences in knowledge that may be necessary to execute certain strategies (Geary, Hoard, Byrd-Craven & DeSoto, 2004; Imbo & Vandierendonck, 2007; Kerkman & Siegler, 1993; Torbeyns, Verschaffel, & Ghesquiere, 2004). For example, in the Siegler & Jenkins (1989) study, increased problem difficulty promoted preschoolers' use of the count-on strategy only among those children who had already demonstrated an ability to execute the strategy on easier problems. Similarly, the differences in the use of decomposition between Chinese and American kindergartners observed in prior work (Geary, Bow-Thomas, Liu & Siegler, 1996) could reflect differences in prior mathematical knowledge and experience. We propose that one aspect of numerical understanding that may contribute to children's use of base-10 decomposition strategies, which are the focus of the present paper, is their understanding of the base-10 structure of the number system.

Base-10 knowledge

There is general agreement that knowledge of the base-10 system is a critical aspect of mathematics performance (Geary, 2006; Miura, 1987; National Research Council, 2001; NCTM, 2000). More specifically, it is widely believed that base-10 knowledge is related to the accuracy of children's computation of multidigit arithmetic problems (Fuson, 1990; Fuson & Briars, 1990; National Research Council, 2001). Errors in carrying and borrowing in written addition problems, for instance, have been attributed to a lack of understanding of base-10 and place value (Brown & Burton, 1978; Fuson, 1990; Hiebert, 1997; Ross, 1986; Valeras & Becker, 1997).

Children learn the structure of the base-10 system and the place-value notation for whole numbers gradually over the course of several years. Even before formal instruction, children are exposed to key aspects of base-10 knowledge. For example, they are exposed to counting beyond 10 and the repeating decade structure of number words. Indeed, the length of children's count string increases between the ages of 4 and 6, which suggests that children increasingly understand the repeating pattern of the decade structure during this time period (Fuson, Richards, & Briars, 1982; Ho & Fuson, 1998; Miller & Stigler, 1987; Siegler & Robinson, 1982). In elementary school, children build on this nascent knowledge through formal instruction about place-value notation, and the properties of the base-10 system.

Children's understanding of the base-10 numeric structure is typically assessed with a block-task (e.g., Miura, Okamoto, Kim, Steere, & Fayol, 1993) in which children are asked to represent two-digit numbers using blocks that include small cubes representing single units and bars that represent ten single units combined together as a collection of ten. Sometime between the ages of 6 and 8, children begin to consistently use base-10 representations (i.e., combinations of ten-bars and single units), rather than single unit cubes, to represent double-digit numerals (Fuson & Briars, 1990; Hiebert & Wearne, 1992; Miura, Okamoto, Kim, Steere & Fayol, 1993; Saxton & Towse, 1998).

We hypothesize that knowledge of the base-10 structure of multi-digit numbers is related to children's use of the base-10 decomposition strategy. A child who has a better understanding of the base-10 structure would also have a better understanding of the composition of numbers in relation to each other and of the fact that numbers can be incremented by intervals other than one (i.e., tens, hundreds), both of which are critical for executing a base-10 decomposition strategy. Consider, for example the addition problem: $23 + 14$. In order to use a decomposition strategy that involves adding tens, then ones, then combining the results, a child must know which digits represent the tens and be able to increment by tens rather than ones. Similarly, better understanding of the base-10 structure may facilitate the use of base-10 decomposition when solving problems that require carryover into the tens place (e.g., $7 + 5 = (7 + 3) + 2 = 12$). A child with poorer base-10 understanding who thinks of multi-digit numbers as collections of single units may use a counting strategy, adding each individual unit (e.g., $7 + 5 =$ “8, 9, 10, 11, 12”).

Based on this analysis, groups who demonstrate greater understanding of base-10 number structure should demonstrate greater use of base-10 decomposition strategies. Indeed, this pattern has been found in studies comparing the performance of American and Chinese first graders. In studies of base-10 number representations, Chinese first graders were more likely than American first graders to represent double-digit numbers using tens-blocks rather than a collection of units (Ho & Fuson, 1998; Miura, 1987; Miura, Okamoto, Kim, Chang, & Steere, 1994; Miura, Okamoto, Kim, Steere & Fayol, 1993). A separate study examining children's arithmetic strategies found that Chinese children in kindergarten through third grade were more likely to use base-10 decomposition strategies in solving addition problems than American students across the same grades (Geary, Bow-Thomas, Liu & Siegler, 1996). Taken together, these two lines of evidence provide support for the hypothesis that base-10 knowledge, as measured by the base-10 block-task, is linked to the frequency of base-10 decomposition use. Importantly, however, no study has investigated this relation by examining both aspects of numeric reasoning within the same group of children; thus, no direct evidence for the existence of a relation between these two aspects of mathematics knowledge exists.

The present study

The present study had three specific aims. The first aim was to examine kindergartners' performance (accuracy and strategy) on single- and mixed-digit addition problems. Earlier research with children of this age has typically involved problems with single-digit addends and focused on the use of counting and retrieval strategies. Including mixed-digit problems in the current study allowed us to examine children's performance on problems with which they had varying degrees of familiarity. Furthermore, unlike earlier research, the current study focused on the frequency and kind of decomposition kindergartners used, as well as counting and retrieval strategies. Motivated by recent findings of a relation between the early use of base-10 decomposition and later mathematics achievement (Carr, Steiner, Kyser & Biddlecomb, 2008; Geary, Hoard, Byrd-Craven & DeSoto, 2004; Geary, Hoard, Nugent & Bailey, 2013), we sought to better understand the range of individual differences in the use of base-10 decomposition already present by kindergarten.

The second aim was to explore the relation between the use of base-10 decomposition and problem characteristics. We hypothesized that base-10 decomposition is among the multiple strategies available to kindergartners, but is not often observed, in part, because they are typically asked to solve addition problems with sums up to 10. On these simple problems, a retrieval or counting strategy may be equally or even more efficient than decomposition; thus, children may choose to execute a counting strategy because they feel more confident in their ability to do so. The difficulty of addition problems varies with the magnitude of addends so that problems involving double-digit numbers are generally more difficult than those involving single-digit numbers and problems involving carryover are more difficult than those without carryover (Frensch & Geary, 1993; Geary, Cormier, Goggin, Estrada, & Lunn, 1993; Green, Lemaire, & Dufau, 2007; Widaman, Geary, Cormier, & Little, 1989). On these more complex problem types, decomposition, particularly base-10 decomposition, would be more efficient, relative to counting strategies. Thus, we predicted that decomposition strategies would be used most frequently on problems involving double-digit addends and those that require carryover.

The third aim was to examine whether the use of decomposition strategies was related to kindergartners' understanding of number structure. Specifically, we investigated (a) the extent of kindergartners' base-10 knowledge and (b) whether individual differences in their base-10 knowledge predict differences in the frequency of using the base-10 decomposition strategy. We hypothesized that children with better base-10 knowledge would be more likely to use the base-10 decomposition strategy on addition problems.

To capture a fuller range of individual differences, we included participants from three nations – US, Taiwan, and Russia – that differ in the linguistic structure of their counting systems and approaches to mathematics instruction. The linguistic structure of counting systems has been posited to be related to base-10 knowledge (Geary, Bow-Thomas, Liu & Siegler, 1996; Miura & Okamoto, 1989). In Asian languages, names of multi-digit numbers transparently reflect the base-10 system (e.g., 12 is “ten-two”). In contrast, English and other Western languages use inconsistent rules and arbitrary number words to express teens and other multi-digit numbers (e.g., 12 is “twelve”). The Russian counting system is mixed — it is more transparent than English and less so than Asian languages. If the use of base-10 decomposition is related to base-10 knowledge, as we hypothesized, then children from these different linguistic backgrounds might vary in their use of the strategy because their counting systems provide different support for base-10 knowledge.

Instructional differences in the emphasis and frequency of exposure to particular strategies may also be related to differences in the use of the base-10 decomposition strategy. Some research indicates that at the elementary grades Chinese teachers tend to place greater emphasis than American teachers on arithmetic strategies that involve decomposing numbers into decades and units (Fuson & Kwon, 1992; Ma, 1999; Perry, 2000; Schleppenbach, Perry, Miller, Sims, & Fang, 2007; Stevenson, Lee, & Stigler, 1986). For example, Fuson & Li (2009) reviewed textbooks widely used in China and the US and found that Chinese textbooks promoted a base-10 decomposition method, whereas American textbooks did not. While there is little information about the nature of instruction across countries prior to first grade, it is possible that similar instructional differences exist. If this is the case, then including kindergartners from different nations had the potential of increasing the likelihood of finding variability in the use of base-10 decomposition.

Several cross-national studies have found differences in first graders' understanding of the base-10 number structure, as measured by the block-task (Miura & Okamoto, 1989; Miura, Okamoto, Kim, Steere & Fayol, 1993), while other studies have reported cross-national differences in children's strategy use (e.g., Geary, Bow-Thomas, Liu & Siegler, 1996; Geary, Fan & Bow-Thomas, 1992). The present investigation provides a bridge between these two bodies of work by testing the hypothesis, in the context of a single study, that children's understanding of base-10 structure predicts their choice of base-10 decomposition strategies. Further, the majority of previous studies compared children from Western countries (primarily, US and Britain) to their peers from East Asian countries (China, Japan, Korea). The present study extended this work to include Russia so as to represent a broader range of backgrounds, in particular with respect to language. The cross-national aspect of this study allowed us to examine whether the relation between the knowledge of base-10 number structure and the use of base-10 decomposition holds despite potential differences in children's cultural/linguistic backgrounds, thus increasing the generalizability of the conclusions.

Method

Participants

The present study was part of a larger cross-national investigation of elementary school students' quantitative development. Participants in the current study included 182 kindergartners from three countries: US, Russia and Taiwan. Table 1 presents the number and characteristics of participants from each country. There were no differences in the percentage of males and females across countries. Russian kindergartners were significantly older than both American and Taiwanese kindergartners, and American kindergartners were significantly older than those from Taiwan (all p's < .001).

Participants were selected to minimize cross-national variability due to sampling from different socio-economic backgrounds (i.e., comparing high-SES students in one country to low-SES students in another country). Equating participants in terms of SES is challenging because countries vary in income criteria used to define SES. Therefore, participants from families with comparable levels of education were recruited. In the US, students were recruited from suburbs largely populated by highly educated professionals (75% of adults with Bachelor's or higher degree, range: 72%–80%). In the other two countries, where educational data were not publicly available, we relied on information about schools' reputation provided by local researchers and officials. Based on this information, students were from school districts with high-status populations (e.g., academics, engineers, physicians, businessmen). Thus, participants across countries were from professional well-educated families, with similar access to educational opportunities.

The US sample was recruited from suburban schools in the state of Massachusetts. The Russian and Taiwanese samples were recruited from schools in the capital cities — Moscow and Taipei, respectively. Testing occurred in the Spring of the school year in all three countries; an analysis of the school calendars indicated that at this time the groups had spent an equivalent amount of time in school during the year.

Table 1
Sample characteristics by country.

	Number of students[a]	Mean age in months (age range)	Female students (%)
USA	89	73 (61–84)	56
Russia	31	82 (68–89)	55
Taiwan	62	75 (68–81)	44

[a] The difference in sample size across countries is due to a limited access to kindergarten students that researchers had in some participating schools. Statistical analyses reported in this paper included homogeneity of variance tests to account for sample size differences.

Participating teachers were asked to complete a survey that included questions about their instructional approach for teaching about place-value, base-10, and arithmetic strategies. Similar to one another, teachers across all countries reported that the base-10 number structure and decomposition strategies were not a focus of instruction in kindergarten. Another similarity across countries was that the educational experiences of the children were varied. In both Taiwan and Russia, no formal curricula exist for kindergarten. In the U.S., the particular curriculum to which children were exposed varied by school.

Materials

Session 1

Materials included two kinds of plastic blocks: 100 small cubes (unit-blocks), each representing a single unit, and 20 longer bars (ten-blocks), with 10 single-unit segments marked. All the blocks were made from the same material and were the same color. In addition, materials included cards (10 cm × 10 cm) with the written numerals that were to be represented with the blocks, as described under Procedure.

Session 2

Materials included two sets of 20 addition problems, each printed on a separate sheet of paper (22 cm × 28 cm). See Appendix A for the list of problems. The two sets (A and B) included the same problems but in different semi-random orders. Both sets began with a block of six single-digit addition problems (three with sums less than or equal to 10 and three with sums above 10), followed by a block of eight mixed-digit problems, followed by another block of six single-digit problems, which was parallel to the first block of problems. This problem order was designed to encourage children by presenting simpler problems at the beginning of the task and to provide relief toward the end of the task after a difficult block of mixed-digit problems.

Procedure

Session 1

During this session, children completed the *block-task* used in previous studies as a measure of base-10 understanding (Miura, 1987). Separate trays with unit- and ten-blocks were placed on the table in front of the child. During the Introduction, the tester explained that the blocks could be used to show numbers. The child's attention was drawn to the fact that there were different kinds of blocks and that a long one was the same as 10 small ones: the tester took 10 unit-blocks from the tray and lined them up against a ten-block while counting from 1 to 10. The introduction was followed by two Practice trials on which the tester presented the child with a number card (e.g., 7) saying, “Now, I'll show you how we can make this number using the blocks.” The tester chose the appropriate number of blocks and placed them next to the card saying, “Look, this shows 7 (pointing to the card) and this shows 7 (pointing to the blocks).” Following Saxton & Towse (1998), who argued that performance on the block-task may vary depending on the numbers (single- or double-digit) used during practice, participants within each classroom were randomly assigned to one of two practice conditions. In one condition, two single-digit numbers were used during practice; in the other condition, a single- and a double-digit number were used on practice trials. Practice was followed by five Test trials. On each test trial, the experimenter presented a child with a different number card and asked the child to show the number using blocks. The five trials included the numbers 12, 16, 28, 34, and 61, presented in a random order for each individual.

The tester recorded how many unit- and ten-blocks the child used to represent each number and made notes about the child's response. Number representations constructed by children were later categorized according to the coding scheme described in Table 2. The coding scheme included seven categories: single unit collection, canonical base-10,

Table 2
Categories of number representations constructed by children.

Category	Description	Examples
Single unit collection	The child uses only single unit-blocks to represent the whole number.	12 = 12 unit-blocks 61 = 61 unit-blocks
Canonical base-10	The child uses the largest possible number of ten-blocks to represent tens and unit-blocks to represent ones.	12 = 1 ten-block + 2 unit-blocks 28 = 2 ten-blocks + 8 unit-blocks 61 = 6 ten-blocks + 1 unit block
Noncanonical base-10	The child uses some ten-blocks (but not the maximum possible quantity) and more than 9 unit-blocks. This strategy can be used only on trials with target numbers 28, 34, and 61.	28 = 1 ten-block + 18 unit-blocks 34 = 2 ten-blocks + 14 unit-blocks 61 = 3 ten-blocks + 31 unit-blocks
Unit confusion	The child does not distinguish between ten- and unit-blocks, either using ten-blocks to represent ones or using unit-blocks to represent tens.	12 = 3 unit-blocks (often one of them is placed separately to represent ten) 12 = 12 ten-blocks (each representing a single unit) 34 = 20 ten-blocks (all available ten-blocks) +14 unit-blocks
Shape-based representation	The child depicts the shape of the numeral with the blocks rather than representing the quantity.	12 = 20 unit-blocks (which are used to "draw" the shapes of 1 and 2) 61 = 13 unit-blocks (to "draw" the shape of 6) and 1 ten-block (to "draw" 1)
None of the above	This category included guessing, no-response, and extremely rare strategies (less than 0.5% of all responses) that did not fit into other categories.	*Guessing*: the child grabs a random number of unit-blocks and places them in a pile, without counting *Rare response*: 28 = 3 ten-blocks (the child covers with a finger 2 single units within one ten-block to arrive at 28)

noncanonical base-10, unit confusion, shape-based representation, and none of the above. The first three categories can be used to correctly represent any double-digit number. Single-unit representations capture both tens and ones in a number as a collection of single units, whereas in canonical base-10 representations, tens and ones are represented separately (with ten-blocks and unit-blocks, respectively), reflecting the base-10 organization of numbers. Noncanonical base-10 representations can be thought of as a hybrid of the first two categories because tens are represented with a mix of ten-blocks and single units. The last three categories listed reflect children's misconceptions about base-10 numeric structure and/or a lack of task understanding.

Session 2

One to two weeks after Session 1 participants were asked to solve a series of addition problems. The tester randomly assigned participants to one of the two orders (A or B) and presented one problem at a time to the child allowing as much time as the child needed to solve the problem before presenting the next one. Participants were not provided with any supplies, such as pencil and paper, but were allowed to use their fingers or count out loud if they wished. The tester observed the child and recorded any overt signs of strategy use (e.g., if the child counted out loud, the tester would mark down a counting strategy). When there were no overt behaviors, the tester asked the participant how he or she "figured it out" after an answer was provided. This kind of combination of behavioral observations and retrospective self-reports has been found to lead to valid strategy classifications (Rittle-Johnson & Siegler, 1999; Siegler, 1987).

Notes and audio-recordings were later reviewed and children's strategies on each problem were coded as one of the main types of strategies described in the introduction: counting, retrieval, fact-based decomposition, base-10 decomposition (see Table 3). An "other" code was also used which included children's reports of guessing or when the strategy was not clear from available behavioral cues and verbal explanations. The "retrieval" code was used only on problems involving single-digit addends, because it has been generally accepted that retrieval applies to stored number facts that typically include single-digit numbers (e.g., Geary, Hoard, Byrd-Craven & DeSoto, 2004). Thus, if a child reported that he/she "just knew" the answer to a mixed-digit problem (e.g., 26 + 8) and no overt behavioral cues were present, the strategy was coded as "other." Children's strategies were coded by three raters, where each child's responses were examined by one rater. The data from 20 participants (11% of the sample) were examined independently by two raters and their agreement rate was 89%. Raters consulted on all instances in which children's reported strategy conflicted with their observed behavior and together agreed on the final code.

Results

In presenting our findings, we begin with the examination of kindergartners' performance on different types of addition problems. As part of this analysis, we investigate children's performance as a function of problem type. We also test for differences in accuracy and strategy choice across participating countries. Second, we examine kindergartners' understanding of numeric structure as revealed by their performance on the block-task, and test the relation between their reliance on base-10 representations on the block-task and the use of base-10 decomposition strategies on addition problems. Age was not correlated to any of the outcome measures, therefore, we collapsed across age for all analyses.

Kindergartners' performance on single- and mixed-digit addition problems

Accuracy

To examine children's addition accuracy, we calculated three scores: (a) overall accuracy, (b) the percentage of problems they attempted, and (c) the accuracy on the problems they attempted (see Table 4). Each outcome provides slightly different information. Overall accuracy provides information about children's overall addition skill; the percentage of problems attempted provides information about their confidence; and percentage accuracy on attempted problems provides information about their ability to accurately execute addition strategies. In all instances, percentages were calculated by averaging each individual child's data and then averaging those statistics across the children within each country or over the whole sample. Table 4 displays these scores by country for three types of problems: single-digit addition with sums up to 10, single-digit addition with sums over 10, and mixed-digit addition. The data suggest that the kindergartners were most accurate on single-digit addition with sums up to 10 and least accurate on mixed-digit addition, but that they seemed about equally willing to attempt the easier and more difficult problems. In other words, children's lower accuracy on mixed-digit problems did not seem to be due to a greater number of "I don't know" responses, but rather to more difficulty accurately executing a strategy on those problems.

Thus, we used overall accuracy as the dependent variable when testing for problem type and country effects. A 3 (Problem Type) × 3 (Country) analysis of variance (ANOVA) on overall accuracy found an effect of Problem Type, $F(2,358) = 76.43$, $p < .001$, $\eta^2 = .30$, and Country, $F(2,179) = 5.26$, $p = .006$, $\eta^2 = .06$, but no interaction, $F(4, 358) = 1.04$, $p = .386$, $\eta^2 = .01$. Pairwise LSD comparisons indicated that in solving single-digit addition problems, kindergartners were more accurate on problems with sums up to 10 than on problems

Table 3
Categories of addition strategies used by children.

Strategy	Explanation	Example
Counting	Starts from number "1" (count-all) or from one of the addends (count-on) and continues to count by enumerating each unit	*Problem*: 4 + 3 Count-all, fingers: Child puts up 4 fingers on one hand and 3 fingers on the other hand, then counts all fingers starting from 1 (1, 2, 3, 4, 5, 6, 7). Count-all, verbal: Child says "1, 2, 3, 4, 5, 6, 7" Count-on, verbal: Child says "5, 6, 7"
Retrieval	Reports knowing the answer and displays no overt counting behavior	This strategy was used for single-digit addition only (e.g., 6 + 2 = 8)
Decomposition		
Base-10 decomposition	Transforms the original problem into two or more simpler problems using the base-10 properties of the number system	(a) "through ten" — transforms addend to sum up to 10 or down from 10, then adds or subtracts remaining part of addend *Problem*: 9 + 5; *Solution*: 9 + 1 = 10; 10 + 4 = 14 (b) "separating tens" — decomposes an addend whose value exceeds 10 into 10 and a remaining portion, carries out addition on tens and units separately before adding them at the end *Problem*: 5 + 13; *Solution*: 13 = 10 + 3, 3 + 5 = 8, 8 + 10 = 18
Fact-based decomposition	Transforms the original problem into two or more simpler problems using previously memorized number facts	(c) "ties" — Decomposes a problem into a problem with identical addends, then adds or subtracts the remaining value *Problem*: 7 + 8; *Solution*: 7 + 7 = 14, 14 + 1 = 15 (d) "learned facts" — Decomposes a problem into a better known problem *Problem*: 5 + 3; *Solution*: 5 + 2 = 7, 7 + 1 = 8
Other	Reports guessing or not knowing, or the strategy cannot be discerned	

with sums larger than 10 ($p < .001$). Further, they were more accurate on both types of single-digit addition problems than on mixed-digit problems, ($p < .001$ and $p = .018$, respectively). In terms of the Country effect, American kindergartners were less accurate across problem types than their peers in Russia and Taiwan ($p = .025$ and $p = .004$, respectively) who were not different from each other.

Strategies

Table 5 presents the percentage of particular strategies that children used to solve different types of addition problems. Overall, kindergartners used a counting strategy most frequently, followed by retrieval and decomposition strategies. The frequency of each strategy, however, appeared to vary by problem type. To test whether and how problem type affected children's strategy choice, we conducted statistical analyses with the percentage of trials on which a particular strategy was used as the dependent variable and with problem type and country as independent variables. Rather than conducting a MANOVA, we carried out ANOVAs separately for each strategy. This decision was based on the fact that not all the strategies were used on every problem type. For example, base-10 decomposition was not used on single-digit addition with sums up to 10, whereas counting was used on this type of problem. Below we present the results for each strategy.

Counting

A 3(Problem Type) × 3(Country) ANOVA showed a significant effect of Problem Type on the percentage of trials on which children chose to use a counting strategy. The effect of Country was not significant but the interaction between Country and Problem Type was, $F(4, 358) = 3.80$, $p = .005$, $\eta^2 = .04$. Tests of simple effects were conducted to better understand the interaction. American students, but not Russian or Taiwanese students, used counting more frequently on single-digit addition with sums over 10, compared to the other types of problems — single-digit addition with sums up to 10 ($p = .019$) and mixed-digit addition ($p = .001$).

Retrieval

In analyzing this strategy, we examined only trials with single-digit problems because, as indicated in Methods, the "retrieval" code was not used with mixed-digit problems. A 2 (Problem Type) × 3 (Country) ANOVA showed a significant effect of Problem Type,

Table 4
Performance on single- and double-digit addition problems.

	Single-digit addition sums ≤ 10% (*SE*)	Single-digit addition sums > 10% (*SE*)	Mixed-digit addition % (*SE*)	All problems % (*SE*)
USA				
Problems attempted	99 (0.6)	98 (0.7)	92 (2.0)	96 (1.0)
Problems correct out of those attempted	80 (3.0)	59 (4.0)	58 (4.0)	64 (3.0)
Problems correct overall	79 (3.0)	59 (4.0)	55 (4.0)	63 (3.0)
Russia				
Problems attempted	99 (5.0)	98 (2.0)	96 (2.0)	98 (2.0)
Problems correct out of those attempted	93 (2.0)	72 (5.0)	70 (5.0)	77 (5.0)
Problems correct overall	93 (2.0)	72 (5.0)	68 (6.0)	76 (5.0)
Taiwan				
Problems attempted	99 (0.5)	97 (2.0)	93 (2.0)	97 (1.0)
Problems correct out of those attempted	89 (3.0)	77 (4.0)	75 (4.0)	79 (4.0)
Problems correct overall	88 (3.0)	75 (4.0)	70 (5.0)	77 (4.0)
Whole sample				
Problems attempted	99 (0.3)	98 (0.7)	93 (1.0)	96 (0.8)
Problems correct out of those attempted	85 (2.0)	67 (3.0)	66 (3.0)	71 (2.0)
Problems correct overall	85 (2.0)	66 (3.0)	62 (3.0)	70 (2.0)

Table 5
Frequency of strategies used on single- and double-digit addition problems.

Strategy	Single-digit addition sums ≤ 10	Single-digit addition sums > 10	Mixed-digit addition
	% of problems (*SE*)	% of problems (*SE*)	% of problems (*SE*)
USA			
Counting	64 (4.0)	70 (4.0)	61 (4.0)
Retrieval	26 (3.0)	5 (1.0)	–
Decomposition	3 (0.8)	11 (2.0)	15 (3.0)
Fact-based	3 (0.8)	5 (1.0)	–
Base-10	–	6 (2.0)	15 (3.0)
Russia			
Counting	52 (7.0)	63 (7.0)	56 (7.0)
Retrieval	41 (7.0)	7 (4.0)	–
Decomposition	4 (2.0)	21 (6.0)	17 (5.0)
Fact-based	4 (2.0)	4 (2.0)	–
Base-10	–	17 (5.0)	17 (5.0)
Taiwan			
Counting	59 (5.0)	59 (5.0)	65 (5.0)
Retrieval	30 (4.0)	15 (4.0)	–
Decomposition	5 (2.0)	17 (4.0)	15 (4.0)
Fact-based	5 (2.0)	5 (1.0)	–
Base-10	–	12 (3.0)	15 (4.0)
Whole Sample			
Counting	60 (3.0)	65 (3.0)	61 (3.0)
Retrieval	30 (2.0)	9 (2.0)	–
Decomposition	4 (0.8)	15 (2.0)	15 (2.0)
Fact-based	4 (0.8)	5 (0.8)	–
Base-10	–	10 (2.0)	15 (2.0)

Note. Trials on which the child's strategy could not be discerned or those on which the child made a guess were coded as “Other” and not included in this table. Statistical tests of differences in the use of strategies by country and problem type are reported in the Results section.

$F(1,176) = 117.86$, $p < .001$, $\eta^2 = .40$. Children in all three countries used retrieval on single-digit addition over 10 less than on single-digit addition within 10. The effect of Country was not significant but the interaction between Country and Problem Types was, $F(2,176) = 5.28$, $p = .006$, $\eta^2 = .06$. Simple effects tests showed that American children used retrieval less frequently than Russian children on single-digit problems with sums up to 10 ($p = .034$) and they also used retrieval less frequently than Taiwanese children on single-digit problems with sums over 10 ($p = .005$).

Decomposition

Participants used both kinds of decomposition: the fact-based approach and the base-10 approach. As shown in Table 5, children used fact-based decomposition only on the two types of single-digit addition problems. A 2 (Problem Type) × 3 (Country) ANOVA with the percentage of *fact-based decomposition strategies* as the dependent variable revealed that neither Problem Type nor Country produced significant main effects or interactions, all p's > .05. The *base-10 decomposition strategy* was used on two types of problems: single-digit addition with sums over 10 and mixed-digit addition. A 2 (Problem Type) × 3 (Country) ANOVA with the percentage of base-10 strategy as the dependent variable showed a main effect of Problem Type, $F(1,179) = 6.03$, $p = .015$, $\eta^2 = .03$, indicating greater use of this strategy on mixed-digit, compared to single-digit, problems. The effect of Country was not significant, $F(2,179) = 1.03$, $p = .359$, $\eta^2 = .01$, nor was its interaction with Problem Type, $F(2,179) = 2.47$, $p = .088$, $\eta^2 = .03$.

Relation between base-10 understanding and base-10 decomposition strategy use

Number representations constructed by kindergartners

To assess children's understanding of base-10, we examined the ways in which they represented double-digit numbers when given the choice to use single unit blocks or tens-blocks. As shown in Table 6, the two most common types of representations across the three countries on the block-task were single-unit and canonical base-10 representations. Non-canonical base-10 representations were used infrequently. Because of the limited use of this category and also because it may represent a transition in thinking about numbers (from a collection of single units to a combination of tens and ones), we focused in subsequent analyses on the two categories that clearly capture distinct ways of representing numbers — single-unit and canonical base-10 representations (henceforth referred to as base-10 representations).

Table 6
Number representation strategies used in session 1.

	Single unit	Canonical base-10	Noncanonical base-10	Other[a]
	% of trials (*SE*)	% of trials (*SE*)	% of trials (*SE*)	% of trials (*SE*)
USA	30 (4)	49 (5)	8 (2)	13 (3)
Russia	22 (7)	63 (8)	8 (3)	7 (5)
Taiwan	19 (5)	57 (5)	7 (2)	17 (4)
Whole sample	25 (3)	54 (3)	8 (1)	13 (2)

[a] The “Other” category includes shape-based representation (5%), unit confusion (3%), and guessing, such as placing a random number of blocks in a pile without counting (5%).

A multivariate analysis of variance (MANOVA) was conducted with Country and Practice Condition as the between-subject variables and with two dependent variables — the percentage of single-unit and base-10 representations (out of all five trials). The results showed no effect of Country on either dependent variable, $F(2, 176) = 2.57$, $p = .079$, $\eta^2 = .03$, and $F(2, 176) = 2.29$, $p = .105$, $\eta^2 = .03$, for unit and base-10 representations respectively. Practice condition did have a main effect on both percentage of single-unit and base-10 representations, $F(2, 176) = 36.89$, $p < .001$, $\eta^2 = .17$, and $F(2, 176) = 19.80$, $p < .001$, $\eta^2 = .10$, respectively. Children in the single-unit practice condition used unit representations more frequently and base-10 representations less frequently than children in the mixed-digit practice condition (see Table 6 for means). There was no interaction between country and practice condition.

Relation between base-10 knowledge and addition strategies

One of the key questions of the present study concerned a possible relation between children's base-10 knowledge (operationalized as their use of base-10 number representations) and their use of base-10 decomposition strategy to solve addition problems. To address this question, we conducted a regression analysis with percentage of Base-10 representations as a predictor and percentage of addition problems on which children used base-10 decomposition as the outcome variable. In order to better understand the nature of the relation between these variables of interest, several other terms were added to the regression model. Specifically, to examine whether the relation held across countries, we added a term capturing a potential interaction between Country and Base-10 representation. We also added Practice condition

Table 7
Hierarchical linear regression: Prediction of base-10 decomposition with base-10 representation, country, and practice condition.

	B	*SE B*	Β
Country	.01	.03	.04
Practice condition	−.02	.03	−.04
Base-10 representation	.23	.09	.44**
Base-10 representation × Country Interaction	−.01	.04	−.06

Note. Country and Practice conditions were entered in the first two steps of the regression, followed by Base-10 representation, followed by the interaction term. The table presents results for the final model.
** $p < .01$.

to determine whether the relation between the variables of interest held when controlling for practice effects (we were unable to include an interaction term for Practice and Base-10 representation because it produced a high level of multicollinearity, $VIF = 16.82$).

The results of the regression analysis are presented in Table 7. The findings confirmed our hypothesis that kindergartners' use of base-10 number representations positively predicted their use of base-10 decomposition strategy. This relation held when controlling for Practice and Country effects. Further, the magnitude of this relation was comparable across the three countries, as evidenced by the lack of interaction between Country and Base-10 representation. This finding was also confirmed by pairwise comparisons showing that the relation between the key predictor (percentage of base-10 representations) and the outcome (percentage of base-10 decomposition) was not significantly different in the three countries (US vs. Russia, $p = .949$; US vs. Taiwan, $p = .885$; Russia vs. Taiwan, $p = .422$).

It is possible that the observed relation between base-10 representations and the use of decomposition strategy was driven by their common link to a third variable. That is, children who have generally better mathematical skills may have used both more advanced number representations (i.e., canonical base-10) and more advanced addition strategies (i.e., base-10 decomposition). In order to address this possibility, we ran a hierarchical regression controlling for children's accuracy on single-digit addition problems with sums less than 10, which served as a proxy for children's basic computational skills. While the computational skills variable was predictive of the dependent variable, its effect decreased after we entered base-10 number representation into the model (Table 8). Importantly, the reliance on base-10 representations accounted for a significant amount of variance, even after controlling for computational skills.

The relation between students' number representations and their strategy choice on the addition task can be further illustrated by comparing the participants who demonstrated a clear preference for a particular representation type on the block-task. Based on the block-task results, we identified two groups – students who relied mostly on base-10 representations versus students who relied mostly on unit representations (at least four trials out of five for either category) – and compared their use of base-10 decomposition strategy for addition. As shown in Fig. 1, students who relied mostly on base-10 representations used more base-10 decomposition strategies ($M = .20$, $SE = .02$) than those who relied mostly on single unit representations ($M = .02$, $SE = .04$). A 2 (Base-10 vs. Unit User Groups) × 3 (USA, Russia, Taiwan) ANOVA indicated a main effect for Group, $F(1, 128) = 12.76$, $p = .001$, $\eta^2 = .09$, but no main effect of Country, $F(2, 128) = .01$, $p = .990$, $\eta^2 < .001$, or interaction with Country, $F(2, 128) = .32$, $p = .727$, $\eta^2 = .01$.

Discussion

Motivated by the findings linking the early use of decomposition to later mathematical achievement (Carr, Steiner, Kyser & Biddlecomb, 2008; Geary, Hoard, Byrd-Craven & DeSoto, 2004; Geary, Hoard, Nugent & Bailey, 2013), the present study investigated individual differences in the use of base-10 decomposition by kindergarten students and the

Table 8
Hierarchical linear regression: prediction of base-10 decomposition with computational ability and base-10 representation.

	B	*SE B*	β
Model 1			
Computational ability	.26	.07	.27**
Model 2			
Computational ability	.16	.07	.16*
Base-10 representation	.17	.04	.32**

* $p < .05$. ** $p < .01$.

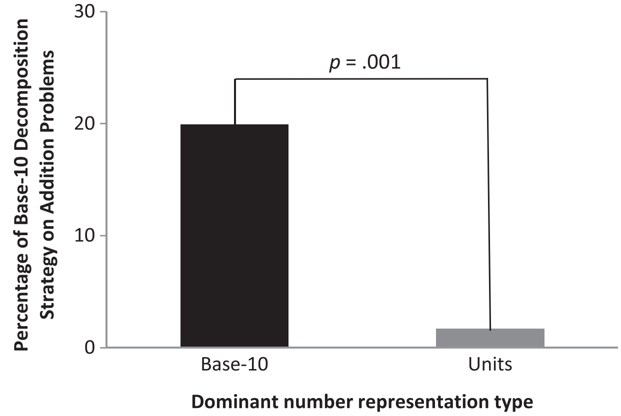

Fig. 1. Use of decomposition strategy by type of number representation $F(1, 128) = 12.76$, $p = .001$, $\eta^2 = .09$.

factors associated with such early differences. Whereas prior studies presented kindergartners primarily with single-digit problems (e.g., Geary, Bow-Thomas, Liu & Siegler, 1996), in the present study they were also asked to solve mixed-digit problems. We reasoned that even though children of this age generally have limited experience with computation beyond single-digit numbers, they may extend the use of available strategies to less familiar problems. In fact, we hypothesized that having to solve mixed-digit addition problems may increase the choice of base-10 decomposition strategy as counting becomes less efficient for this type of problem. Further, we hypothesized that children's growing understanding of the base-10 numeric structure may facilitate their use of base-10 decomposition in solving mixed-digit problems. Below we consider our findings related to these hypotheses and discuss their potential implications.

Kindergartners' performance on single- and mixed-digit addition

Accuracy

Kindergartners in the present study displayed a uniformly high level of confidence, attempting to solve the vast majority of both single- and mixed-digit problems with which they were presented. Their accuracy rates varied across problem types as expected, with the highest accuracy observed on single-digit addition without carryover, and the lowest accuracy — on mixed-digit addition.

While kindergartners' accuracy was lowest on mixed-digit problems, they solved more than half of these problems correctly. This result was true even of American kindergartners who were less accurate across all problems than their Russian and Taiwanese peers. This finding has two important implications. First, it indicates that kindergartners have greater addition knowledge than suggested by previous research that included only problems with single-digit addends. Secondly, it suggests that despite minimal to no formal instruction on how to solve these kinds of problems children are able to extend prior knowledge to less familiar, more challenging problems. Relevant prior knowledge may have included familiarity with the counting sequence, understanding of number composition and numeric relations, and – importantly – access to a variety of addition strategies.

Strategies

The Overlapping Waves model of development posits that at any given point in development children may have a predominant strategy, but also know and be able to use multiple strategies depending on the situation. The current findings provide further evidence for this perspective and strategy choice in arithmetic. In this study, like in other studies, kindergartners' predominant strategy was counting, but this strategy was not used exclusively. The kindergartners also used retrieval and decomposition to solve some problems. This variability in strategy use was found both across all the problems as well as within problem types.

Consider, for example, children's use of retrieval on problems with single-digit addends in this study: kindergartners used retrieval less frequently on problems with sums over ten than on problems with sums up to ten. Theoretically, retrieval is equally efficient for both kinds of single-digit problems. On the other hand, kindergartners have less experience memorizing number facts with sums above ten because kindergarten instruction, across countries, tends to focus on number facts up to 10. Thus, children's choice to use retrieval more frequently on the problems with sums up to 10 than on problems with sums greater than 10 probably reflects adaptive choice based on their judgment of their ability to do so accurately (Siegler & Shrager, 1984).

Children's use of decomposition is also consistent with the Overlapping Waves perspective. The kindergartners used the base-10 decomposition strategy relatively infrequently. The Overlapping Waves model suggests that this is because base-10 decomposition is an emerging strategy that has not yet gained prominence with most kindergarten-aged children, but may replace counting as the dominant strategy with age, experience, and explicit instruction in different strategies. Indeed, our data suggest that base-10 decomposition may become more dominant as children are faced with more difficult problems and as they acquire greater knowledge about the base-10 number structure.

Somewhat surprisingly, given previous findings of cross-national differences in first graders' base-10 knowledge and use of decomposition, we found no differences across countries in kindergartners' use of base-10 decomposition. It should be noted that two other studies that also included children younger than first grade found no cross-national differences in strategy use (Wang & Lin, 2005; Zijuan & Chan, 2005). This lack of differences may be due in part to a greater similarity in instruction across countries in preschool and kindergarten than later grades. Literature suggests that base-10 decomposition is taught more explicitly in Asian countries than in the US starting in first grade (e.g., Fuson & Li, 2009; Perry, 2000); however, there is less evidence of instructional differences at the kindergarten level. The teacher survey used in our study indicated more similarity than differences across countries, with little instruction on base-10 strategies at kindergarten across all countries. Our findings suggest that until they receive exposure to explicit strategy instruction, children from different backgrounds follow a similar path in strategy discovery and use — a possibility that could be tested in future research though a study including both kindergartners and first graders.

Factors related to kindergartners' use of base-10 decomposition

Problem difficulty

The difficulty of a problem influences strategy choice (Siegler & Shrager, 1984). One reason is that it influences the efficiency of a given strategy. For example, counting-on can be more efficient than decomposition on problems with a small second addend (e.g., 18 + 1). Similarly, retrieval is generally more efficient than decomposition for solving single-digit addition problems, especially those with sums up to ten. On simple problems, decomposing numbers would likely take more time and effort than retrieving a number fact. A second reason is that problem difficulty influences the likelihood of executing a strategy accurately. For example, the sums to problems with large addends are unlikely to be easily retrieved from memory, due to infrequent practice with them.

These points suggest that decomposition would be used least on single-digit problems with sums up to 10 and most on mixed-digit problems. Indeed, this was the pattern found in the current results. Similar to prior studies (e.g., Geary, Bow-Thomas, Liu & Siegler, 1996; Siegler, 1987), our participants' use of decomposition strategies was minimal on single-digit addition problems with sums up to 10 (4% of problems). Yet, their use of these strategies increased (15% of problems) when they were presented with more complex addition problems involving carryover and/or double digits. These results indicate that children know and can use decomposition by the end of kindergarten, but are unlikely to use decomposition unless the problem difficulty makes it more advantageous than their more predominant approaches—either because it is more efficient or leads to greater accuracy on more difficult problems. Similarly, a study of 4- and 5-years olds' addition strategies found that with practice the children were able to "discover" the strategy of counting-on from the larger addend (Siegler & Jenkins, 1989). After this insight, however, they showed little generalization of the strategy — continuing to rely predominantly on the laborious count-all strategy for problems with addends smaller than five. To promote generalization, Siegler and Jenkins presented the children with challenge problems, such as 3 + 22, that included a small addend and a very large addend. After encountering the challenge problems, generalization of the count-on strategy increased, such by the end of the experiment it became the dominant strategy. This generalization only occurred, however, for children who were already able to use the strategy to some extent on simpler problems. Thus, presenting complex problems that highlight the efficiency of more an advanced strategy can hasten children's consistent use of it, but only if children possess the ability to execute it. Children who are not yet able to use the strategy might first need explicit practice on how to use the new strategy with simpler problems before generalization can occur to more complex ones.

Base-10 knowledge

Knowledge of relevant skills is necessary for both learning new strategies, as well as executing them correctly. For example, to use a count-all strategy successfully, a child needs to know the counting sequence starting from one, but to use a count-on strategy successfully a child must also know how to start counting from a number other than one. A decomposition strategy requires even more sophisticated skills, such as knowing that a number is the sum of smaller numbers and being familiar with the ways in which numbers can be decomposed (e.g., 8 can be 4 + 4 or 5 + 3 or 6 + 2, etc.). Our finding that children relied on counting strategies more frequently than decomposition strategies across problem types may be due to the fact that kindergartners have more experience with the counting sequence than with number facts that are required for both retrieval and decomposition strategies.

In order to use a base-10 decomposition strategy in particular, children have to be able to decompose numbers using base-10 properties of the number system. Thus, one aim of the present study was to examine the extent to which kindergartners possess base-10 knowledge and, if so, whether this knowledge contributes to the frequency with which they use base-10 decomposition. The results indicate that by the spring of kindergarten, children (at least those from highly educated families) do have some understanding of the base-10 structure of the number system: kindergartners used base-10 representations on the block-task on more than half of the trials and more often than single-unit representations. This level of understanding was surprising given that teachers in all three countries reported that base-10 was not a focus of instruction in kindergarten, raising questions about what kinds of experiences contributed to this knowledge.

The present study included children from US, Taiwan, and Russia in hopes of capturing greater variability in base-10 knowledge, which could potentially lead to differences in base-10 decomposition. This choice was based on prior research showing cross-national differences in base-10 knowledge — specifically, research which found that Asian children used substantially more base-10 number representations on the block-task than American children and proposed that this was due to differences in the transparency of the counting systems (e.g., Miura, 1987; Miura & Okamoto, 1989). In this study, however, there were no between-country differences in the percentage of trials on which children used base-10 representations; kindergartners from all three countries were equally likely to use base-10 representations and to use them more often than single-unit ones. Previous studies with first graders may have conflated language and instructional experience. While the kindergartners in our study came from different language backgrounds, our survey of teachers suggested their instructional experiences were similar. Thus, just as with the findings regarding strategy use, we suspect that the difference in the cross-national findings of base-10 representations between older and younger children may be due to the equally little amount of formal instruction they have received in kindergarten and instructional differences across the countries in more advanced grades.

Most important for the purpose of the present study is the finding that kindergartners' understanding of the base-10 structure of the number system is an important contributor to how frequently they choose to use a base-10 decomposition strategy. The percentage of trials on which children used a base-10 representation on the block-task predicted the percentage of addition problems on which they used a base-10 decomposition strategy. Furthermore, the relation between base-10 knowledge, as measured by representation type on the block-task, and base-10 decomposition held even after controlling for other computational skills. In fact, the use of base-10 representations was a stronger predictor of the use of base-10 decomposition strategy on multi-digit addition problems than computational accuracy on single-digit addition problems, indicating that this predictive relation does not simply reflect general math ability. Further, our data point to the universality of this relation: children's use of base-10 representations was equally predictive of their use of the base-10 decomposition strategy across the three countries in our sample, despite potential differences in mathematical experiences and the structure of the counting systems.

Suggestions for educational practice

The current findings are consistent with the theoretical premise that greater base-10 knowledge leads children to rely on a base-10 strategy more frequently. From this perspective, a greater focus on the base-10 number structure at the early stages of math instruction may be particularly beneficial for the development of computational skills required to solve problems with multi-digit numbers. It also suggests that the effectiveness of explicitly teaching children to use base-10 decomposition may, in part, depend on the extent of their base-10 knowledge. These suggestions are consistent with the strong emphasis on base-10 and place value concepts found in the Common Core Mathematics Standards for the primary grades (National Governors Association, 2010). They are also consistent with a study by Agodini et al. (2009) that examined the benefits of different first grade curricula on students' math achievement and found the greatest benefits for curricula with an emphasis on base-10 knowledge.

The associational nature of this study, however, does not allow for a strong claim of a causal relation between base-10 knowledge and the frequency of use of a base-10 decomposition strategy. Now that the link has been established, follow-up studies testing the theorized direction of the relation would be worthwhile. A longitudinal study assessing whether base-10 knowledge at preschool or kindergarten predicts the frequency of use of base-10 decomposition in first or second grade would provide stronger evidence. Additionally, an experimental study could be conducted. If kindergartners randomly assigned to receive instruction related to base-10 number structure later used base-10 strategies more frequently than those whose receive an equivalent amount of instruction in some other aspect of mathematics, than a causal conclusion could be made.

A second implication of these findings is that children might benefit from being introduced to mixed-digit addition problems in kindergarten. The kindergarten children from all three countries included in the present study readily attempted almost all (>90%) of the mixed-digit problems presented and were able to solve more than half of them correctly, even though teachers reported that they were not exposed to this problem type in the classroom. Thus, children may be ready to be introduced to multi-digit addition earlier than typically occurs in instruction. Of course, it is important to acknowledge that our participants were from middle- and upper-middle-class families, and children from lower-SES backgrounds, regardless of country, are likely to show lower levels of computational skills (Griffin, Case, & Siegler, 1994; Jordan, Huttenlocher, & Levine, 1994). Nevertheless, the current results suggest that providing children with opportunities to encounter and attempt solving these more difficult types of problems offers them opportunities to increase their use and improve their mastery of decomposition strategies. Together the results suggest that early use of base-10 decomposition may benefit from instruction on relevant skills, such as base-10 knowledge, and exposure to problems that elicit its use and highlight its efficiency.

Appendix A. List of problems used in session 2

Single-digit addition with sums up to 10	Single-digit addition with sums above 10	Mixed-digit addition
2 + 6	3 + 8	5 + 22
5 + 4	8 + 7	18 + 3
3 + 4	6 + 5	37 + 2
6 + 3	5 + 7	5 + 59
4 + 2	4 + 9	26 + 8
3 + 7	9 + 8	6 + 41
		4 + 38
		15 + 3

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