MIND, BRAIN, AND EDUCATION

Mathematics Teacher Educators' Perceptions and Use of Cognitive Research

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ABSTRACT— Instructors ($N = 204$) of elementary mathematics methods courses completed a survey assessing the extent to which they value cognitive research and incorporate it into their courses. Instructors' responses indicated that they view cognitive research to be fairly important for mathematics education, particularly studies of domain-specific topics, and that they emphasize topics prominent in psychology studies of mathematical thinking in their courses. However, instructors reported seldom accessing this research through primary or secondary sources. A mediation analysis indicated that mathematics methods instructors' perception of the importance of the research predicts their incorporation of it in their courses, and that this relation is partially mediated by their accessing of it. Implications for psychologists who have an interest in education and recommendations for facilitating the use of cognitive research in teacher preparation are discussed.

Over the past two decades, there has been increased interest in the application of cognitive science research for improving instruction (Bransford, Brown, & Cocking, 1999; Darling-Hammond, 2010; Newcombe et al., 2009). Numerous sources describe the implications of cognitive research for the instruction of reading, science, and mathematics (e.g., National Mathematics Advisory Panel, 2008; Pashler et al., 2007; Rayner, Foorman, Perfetti, Pesetsky, & Seidenberg, 2001; Siegler, 2003). However, the extent to which cognitive research actually influences teachers' instruction is largely unknown.

This study investigated this question in relation to the training of elementary mathematics teachers. The main goals of the study were to examine teacher educators' perceptions of the importance of cognitive psychology research and particular topics, their use of cognitive psychology research in elementary mathematics methods courses in which preservice teachers learn specifically about mathematics pedagogy and student learning, and potential factors related to the incorporation of cognitive research in mathematics education coursework. This information can inform researchers' choice of research questions and dissemination venues, which, in turn, could facilitate a stronger connection between cognitive science and educational practice.

POTENTIAL OF COGNITIVE PSYCHOLOGY TO INFORM ELEMENTARY MATHEMATICS TEACHING

The premise that cognitive research can be used to improve mathematics instruction is generally based on two assumptions: (1) psychology research generates information that has implications for mathematics education and (2) teachers with knowledge of this research will be better equipped to improve children's mathematics understanding than teachers without this knowledge.

Psychological Studies of Mathematical Thinking

There is now considerable knowledge from cognitive science about how children learn mathematics (Geary, 2006; Siegler, 2003). Our review of the literature in this area suggested four general categories of research that might be particularly useful to elementary mathematics teachers' pedagogy. Table 1 presents these categories and provides illustrative studies for each.

The first two categories of research can potentially help teachers determine how to sequence and differentiate instruction as well as assess their students. The first of these categories—*Developmental Progression and Common Misconceptions*—involves research about the developmental progression

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Table 1
Four Categories of Cognitive Psychology Research That Are Relevant to Elementary Mathematics Education and Illustrative References

Thematic category	*Illustrative references*
Developmental progression and common misconceptions of key mathematics skills and concepts	Carpenter and Moser (1984) Gallistel and Gelman (1992) Laski and Siegler (2007) Mix, Levine, and Huttenlocher (1999)
Cognitive strategies for mathematical problem-solving	Geary et al. (2004) Hallett, Nunes, and Bryant (2010) Siegler (1988)
Cognitive processes involved in the acquisition of mathematics skills and concepts	Imbo and Vandierendonck (2007) Klein and Bisanz (2000) McNeil and Alibali (2004)
Influences of instructional techniques on the cognitive processing of mathematics	Goldin-Meadow, Cook, and Mitchell (2009) Rittle-Johnson (2006)

of children's understandings of mathematics concepts. For example, Siegler and colleagues have demonstrated that children initially represent the magnitude of numbers logarithmically, such that they exaggerate differences between numbers at the low end of the numerical scale and minimize differences at the high end. With age and, presumably, greater experience with larger numbers, children's representations of numerical magnitude become increasingly linear (Laski & Siegler, 2007; Siegler & Booth, 2004). The second category—*Cognitive Strategies for Problem Solving*—involves research about the strategies children use in mathematics, the value of those strategies, and the dimensions by which children select among them. For instance, cognitive psychology research has demonstrated that at any given time in development children know and use multiple addition strategies and that children's selection of strategies depends on multiple factors such as problem difficulty and confidence level (Ashcraft, 1992; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Shrager & Siegler, 1998).

The next two categories of research can potentially help teachers understand individual differences in rate of learning and how to design instruction that facilitates learning. The first of these categories—*Cognitive Processes Involved in the Acquisition of Mathematics Skills*—involves research about the general cognitive processes involved in mathematical thinking and the acquisition of mathematical knowledge. For example, McNeil and Alibali (2004) examined how encoding—a cognitive process by which stimuli are mentally represented—influences arithmetic performance. They found that elementary students who encode key features of problems and instructional examples are more likely to employ accurate solution strategies and solve problems correctly. The last category—*Influences of Instructional Techniques on the Cognitive Processing of Mathematics*—involves research about the influence of particular instructional techniques on cognitive learning processes. For example, self-explanation has been found to promote the acquisition of more sophisticated mathematical concepts (e.g., Rittle-Johnson, 2006; Stigler & Hiebert, 1999).

Knowledge Important for Teaching

Knowledge about the topics predominant in cognitive psychology studies of mathematical thinking is considered an important aspect of the knowledge required to teach mathematics effectively (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Shulman, 1987). Knowledge of mathematics content, while important, is not a sufficient foundation for the effective teaching of mathematics (National Research Council, 2010). In addition to content knowledge, pedagogical-content knowledge—knowledge related to how to teach particular content—is considered to be fundamental to teachers' effectiveness (Ball & Bass, 2000; Darling-Hammond & Bransford, 2005; Shulman, 1987). An important aspect of current conceptions of pedagogical-content knowledge is knowledge of students' cognitive strategies, learning processes, and prior knowledge that might be either facilitative or inhibitive for the particular learning task at hand (Ball et al., 2008; Hill, Rowan, & Ball, 2005).

Indeed, there is causal evidence that increasing teachers' knowledge of developmental progressions and learning processes improves their capacity to plan and deliver mathematics instruction as well as to analyze student errors. When elementary school teachers are provided with professional development designed to improve their understanding of children's mathematical thinking, their instruction changes in ways that are related to higher student achievement (Fennema et al., 1996). Similarly, Saxe, Gearhart, and Nasir (2001) found that professional development that included student thinking and learning processes led to greater effects on upper elementary students' conceptual understanding of fractions than did an equivalent amount of professional development focused on general pedagogical techniques.

RESEARCH USE IN EDUCATION

Previous findings about research utilization indicate, at best, inconsistent use of empirical research in education (Broekkamp & van Hout-Wolters, 2007; Hemsley-Brown & Sharp, 2003; Huang, Reiser, Parker, Muniec, & Salvucci, 2003; Nelson, Leffler, & Hansen, 2009). Two factors have emerged as barriers to the use of research: perceived importance and comprehensibility.

Perceived Importance

Qualitative studies consistently report that a primary factor influencing educators' use of research findings is its perceived importance (Broekkamp & van Hout-Wolters, 2007; Nelson et al., 2009). Teachers generally perceive research to lack applicability and, therefore, are unlikely to seek out and access research articles (Hemsley-Brown & Sharp, 2003). Teachers also respond most positively to research that identifies specific pedagogical strategies and techniques as opposed to articles that simply document research findings (Zeuli, 1994). These results suggest that unless empirical research is seen as having direct implications for education and teaching, it is unlikely to be accessed and utilized by educators.

Perceived Comprehensibility

The perceived comprehensibility of research also seems to be related to the utilization of research (Nelson et al., 2009). Educators have reported that the complexity and theoretical orientation of research is problematic (Huang et al., 2003). Educators also report that the use of jargon, technical-language, and statistics make research findings difficult to interpret and, therefore, not useful (Hemsley-Brown & Sharp, 2003; Nelson et al., 2009). These findings suggest that the utilization of psychological research in education might be particularly problematic given differences in terminology, methodology, and presentation styles between the two fields.

TEACHER EDUCATION PROGRAMS AS AN INTERMEDIARY IN THE USE OF PSYCHOLOGICAL RESEARCH

One important role of teacher preparation programs is to communicate research-based knowledge to teachers (Cochran-Smith, 2005). Current credentialing guidelines reflect the assumption that providing teachers with knowledge of child development, learners, and learning is an essential aspect of an effective teacher preparation program (NCATE, 2008).

Within teacher education programs, methods courses play a central role in teacher training. These courses are designed specifically to instruct preservice teachers about how to teach academic content. An elementary mathematics methods course usually covers various topics, including pedagogical approaches that promote children's mathematical thinking and development, how elementary students learn mathematics, and assessment techniques (Van de Walle, Karp, & Bay-Williams, 2009). In the design of their courses, instructors likely reflect upon and make instructional decisions about the topics and research most important to mathematics instruction.

Because methods courses emphasize practical pedagogical approaches, research presented in these courses may be more meaningful to preservice teachers than research presented in other courses, such as stand-alone psychology courses (Ball & Cohen, 1999; Darling-Hammond, Wei, & Orphanos, 2009). Furthermore, these courses likely provide more opportunity to discuss the implications of the results for teaching and to embed the findings in the practice of teaching, similar to the professional development approaches that have been successful (Fennema et al. 1996). Thus, incorporating cognitive findings into methods courses may increase the likelihood that the findings will affect preservice teachers' later instruction.

THE CURRENT STUDY

The above analysis suggests that instructors of elementary mathematics methods courses in teacher education programs are an important source for determining which topics may be most relevant to improving current mathematics instruction and most useful for elementary mathematics teachers to know. In this study, instructors were asked to rate the importance of cognitive psychology, in general, and of specific topics to the preparation of future elementary mathematics teachers. Instructors were also asked how much they emphasized various topics in their elementary mathematics methods courses.

The above analysis also suggests that these instructors may play a particularly important role in communicating cognitive findings about mathematics learning to teachers of elementary mathematics. We expected perceived importance to predict accessing and incorporation: instructors who perceived cognitive research to be more important would be more likely to access it and incorporate it in their courses. We also hypothesized that the more instructors accessed this research, the more likely they would be to incorporate into their courses.

Last, we predicted that perceived comprehensibility would moderate the relations between perceived importance and accessing and between accessing and incorporation. Instructors who perceive cognitive psychology research to be difficult to understand, despite being potentially important, might be less inclined to believe that reading the work will be a fruitful use of time, thus may be less likely to access it. Further, instructors who do access the research, but perceive it to be difficult to understand, might feel less confident in their own understanding of the work, their ability to explain the work, and their students, potential to understand the work, and thus may be less likely to incorporate it into their courses.

METHOD

Participants

Participants were instructors of elementary mathematics methods courses ($N = 204$) from 195 institutions within the continental United States, representing 46 states and the District of Columbia. Participants were recruited from a range of universities who were members of the American Association of Colleges of Teacher Education (AACTE). The recruitment email invited instructors to participate in a study about "what teachers of elementary mathematics methods believe our nation's teachers should know and be able to do"—there was no mention in the recruitment email about the study being about instructors' views on research.

The majority of participants were tenure-track professors: assistant (38.4%), associate (29.5%), and full (17.9%). A small percentage of the instructors (14.2%) held nontenure track positions. The majority of participants (74.72%) indicated that they had complete control of their course design and syllabus.

Survey Development and Content

Data were collected using an online survey. Because there was no existing instrument that could be used to address the current study's research questions, the research team constructed a survey through an iterative, collaborative process. In addition to our own revisions to multiple drafts of the survey, it underwent both substantive and methodological reviews by discipline experts not affiliated with the research project. The final survey reflects all changes suggested by these external reviewers.

The survey was designed to examine four constructs related to cognitive research, namely instructors' (1) perceptions of its *importance* to mathematics education and the preparation of teachers; (2) perceptions of its *comprehensibility*; (3) *accessing*; and (4) *incorporation* in their elementary mathematics methods course. These data generally took the form of responses to rating scales. For each construct, we included various survey items tapping different aspects of the construct such that scales or indexes could be created. The reliability and internal structure of our measures are reported in the results section.

For some questions, a "Don't know or unsure" response option was included in order to discriminate individuals with neutral opinions (e.g., "neither agree nor disagree") from those who did not understand the question or item content. "Don't know or unsure" responses were set as missing data for all inferential analyses.[1] For all items, participants were asked to respond with regard to "research with implications for elementary mathematics education."

RESULTS

Perceptions and Use of Cognitive Research

Perceived Importance

Instructors' responses indicated that they generally view cognitive and developmental studies of mathematics to be somewhat important for mathematics education, but that this sentiment seems to be stronger for particular mathematics-related topics than the research as a whole. When asked to indicate their agreement on a 5-point scale with the statements "[Cognitive *or* developmental] psychology research findings are applicable to mathematics education" ($\alpha = .89$) and the statements "[Cognitive *or* developmental] psychology research addresses topics, issues, and problems that are important to elementary mathematics education," ($\alpha = .87$) the mean responses fell between "neither agree nor disagree" and "agree," (3.79, $SD = 0.74$ and 3.65, $SD = 0.69$, respectively).

As shown in Table 2, instructors' responses were more favorable, however, when asked to rate the importance of specific topics to the preparation of future elementary mathematics teachers, particularly in relation to domain-specific topics. The mean of the three domain-specific items was significantly higher than the mean of the three domain-general items, $t(185) = 10.68$, $p < .001$.

Perceived Comprehensibility

In general, the data indicated that instructors perceived cognitive and developmental psychology research to be somewhat comprehensible. The mean agreement with the statements "[Cognitive *or* developmental] psychology research findings are accessible (i.e., capable of being understood)" ($\alpha = .87$) fell between "neither agree nor disagree" and "agree" (3.54, $SD = 0.79$).

Accessing

To collect information about the extent to which instructors' access research, we first asked participants to rate their agreement with statements regarding the extent to which they (a) "keep abreast of empirical research" (b) "read empirical research articles to access research," and (c) "read practitioner-friendly articles to access summaries of research" from the field of mathematics education, cognitive psychology, developmental psychology, and educational technology. We averaged individuals' responses across the three items for each field; thus, the data do not discriminate between whether instructors access research findings from primary or intermediary sources. Reliabilities for the item triads (one per field) were high (α's ranging from .82 to .91).

Instructors' responses suggested that they read research findings from various fields. As would be expected, they reported accessing relevant research from mathematics education more than research from other fields. The mean response across the three agreement items for mathematics education was between "agree" and "strongly agree" ($M = 4.46$, $SD = 0.67$), whereas the means for the other fields were around "neither agree or disagree": educational technology was 3.20 ($SD = 1.09$), cognitive psychology was 3.02 ($SD = 0.98$), and developmental psychology was 2.79 ($SD = 0.93$).

Table 2
Instructors' Ratings of Importance of Specific Topics to the Preparation of Future Teachers of Elementary Mathematics

	Percentage of instructors in each rating category					
Instructional topic	*Not at all important (%)*	*Somewhat important (%)*	*Important (%)*	*Very important (%)*	*Don't know/unsure (%)*	*Mean rating (SD)*[a]
Basic cognitive processes (e.g., memory, attention)	0.00	15.76	58.15	23.91	2.17	3.08 (.63)
Basic learning processes (e.g., encoding, generalization, automaticity)	2.15	19.89	51.61	23.66	2.69	2.99 (.73)
Cognitive strategies and processes	0.00	5.91	45.70	48.39	0.00	3.42 (.60)
Common mathematical misconceptions	0.00	5.91	37.10	56.99	0.00	3.51 (.61)
Common developmental sequences in mathematics	0.00	4.84	46.24	48.39	0.54	3.44 (.59)
The ability to predict differences in students' performance/problem solving based on students' prior knowledge	0.00	15.59	48.92	34.95	0.54	3.19 (.69)

[a]"Don't know or unsure" responses were set as missing for the calculation of means and standard deviations.

Next, we asked participants to rate their familiarity with cognitive researchers who conduct research directly relevant to mathematics education. Half, or more of the instructors, reported being "not at all familiar" with each of these researchers' work. For instance, 72% and 49% of instructors were not at all familiar with the work of David Geary and Robert Siegler, respectively—two cognitive psychologists assigned to the 2008 National Mathematics Advisory Panel because of the importance of their work to mathematics education.

Last, we examined whether instructors tend to access psychology findings through primary sources by asking them to rate the frequency with which they "read or reference articles or information" from six well-respected cognitive/developmental psychology journals whose impact factors ranged from 1.36 to 3.77 in 2010. As shown in Table 3, the mean responses for all these journals fell between "never" and "seldom."

Incorporation Into Elementary Mathematics Methods Course

The data indicate that topics prominent in cognitive psychology studies of mathematics are incorporated to some extent in elementary mathematics methods courses. When asked to estimate the percentage of time they allocate to various topics within their methods course, instructors' estimates indicated that the greatest proportion of time ($M = 25.48\%$, $SD = 12.95\%$) is spent discussing specific pedagogical techniques, methods, or activities. However, instructors reported allotting between 11% and 15% of the total course time on topics prominent in cognitive studies of mathematics: the developmental sequence of mathematics concepts and skills ($M = 13.34\%$, $SD = 10.20\%$), cognitive strategies ($M = 15.13\%$, $SD = 8.32\%$), and mechanisms and processes underlying mathematics learning ($M = 11.10\%$, $SD = 6.39\%$). Similarly, as shown in Table 4, when asked to report how much they emphasized various topics in their course on a 4-point rating scale ranging from "not at all" to "to a great extent," mean responses fell between "to some extent" and "to a great extent" for topics related to cognitive research, with the developmental sequence of math concepts and skills being emphasized the most ($M = 3.48$, $SD = 0.61$).

The data presented in Table 5 indicates that when instructors were asked specifically about the extent to which they discussed various topics when teaching about arithmetic, a key topic in most elementary mathematics methods courses (cf. Van de Walle et al., 2009) for which there is a great deal of relevant cognitive research, mean responses fell between "to some extent" and "to a great extent" for topics for which there is relevant cognitive research. Instructors' mean responses for domain-general cognitive processes, however, fell between "very little" and "to some extent" when they were asked what they would discuss in relation to a hypothetical student arithmetic error.

Factors Related to Instructors' Incorporation of Research Into Elementary Mathematics Methods Courses

To examine potential factors related to instructors' incorporation of cognitive research into their courses, we constructed a composite variable for each of the four key constructs. We then conducted correlation, regression, and mediation analyses to examine the relations among the four variables.

Variable Construction

Variables were constructed from multiple survey items. The survey items were written to tap each of the constructs;

Table 3
Frequency With Which Instructors Reported Reading or Referencing Work From Specific Psychology Journals

	Percentage of instructors in each rating category				
Journal title	*Never (%)*	*Seldom (%)*	*Sometimes (%)*	*Frequently (%)*	*Mean rating (SD)* [a]
Child Development	44.57	33.14	18.86	3.43	1.81 (.86)
Cognition and Instruction	36.00	28.00	28.00	8.00	2.08 (.98)
Cognitive Development	54.60	30.46	11.49	3.45	1.64 (0.82)
Developmental Psychology	68.39	28.16	2.87	0.57	1.36 (0.57)
Journal of Applied Developmental Psychology	67.05	24.89	7.51	0.58	1.42 (0.66)
Journal of Educational Psychology	48.57	33.14	14.29	4.00	1.74 (0.85)

[a]"Don't know or unsure" responses were set as missing for the calculation of means and standard deviations.

Table 4
Emphasis on Various Topics in Most Recent Elementary Mathematics Course Taught

	Percentage of instructors in each rating category					
Topic	*Not at all (%)*	*Very little (%)*	*To some extent (%)*	*To a great extent (%)*	*Don't know/unsure (%)*	*Mean rating (SD)*[a]
Developmental sequence of mathematics concepts and skills	0.00	5.98	40.22	53.80	0.00	3.48 (.61)
Eliciting and assessing student mathematical thinking	0.00	0.54	35.68	63.78	0.00	3.63 (.50)
Cognitive strategies for mathematics problem solving	0.54	1.61	40.32	57.53	0.00	3.55 (.56)
Mechanisms and processes underlying mathematics learning	1.08	7.53	53.76	37.63	0.00	3.28 (.65)
Differences between typical and atypical learners in mathematics	3.24	22.70	54.59	18.92	0.55	2.90 (.74)

[a]"Don't know or unsure" responses were set as missing for the calculation of means and standard deviations.

however, the final selection of items used for variable construction was data-driven (e.g., factor analysis). The items comprising each composite variable can be found in Table 6. The internal consistency (α) of the ratings used to construct the *perceived importance* variable was 0.76. Principal components analysis (PCA) revealed a single component solution with an eigenvalue of 2.77 that explained 46% of the total variance (component loadings ranged from 0.52 to 0.83). The internal consistency (α) of the two agreement items used to construct the *perceived comprehensibility* variable was .87. PCA showed a single component solution with an eigenvalue of 1.77 that explained 88% of the total variance, with item component loadings at 0.94. For the *accessing* variable, we found a single major factor with an eigenvalue of 7.35 using principal axis factoring (unrotated) that explained 41% of the common factor variance. Factor loadings for this first factor ranged from 0.33 to 0.79. For the *incorporation* variable, common factor analysis (unrotated) revealed a single major factor with an eigenvalue of 2.64 that explained 22% of the variance. The factor loadings for this first factor ranged from 0.36 to 0.67.

Correlation and Regression Analyses

Incorporation was related to both perceived importance ($r = .60, p < .001$) and accessing ($r = .50, p < .001$), but was not related to perceived comprehensibility. Perceived importance and comprehensibility were both related to accessing ($r = .30$, $p < .001$ and $r = .23, p < .01$, respectively).

A regression analysis indicated that both perceived importance and perceived comprehensibility predicted the extent to which instructors accessed cognitive psychology research, $F(2, 122) = 8.87$, $p < .001$. Together, however, these factors only accounted for 11% of the variance in instructors' accessing of cognitive psychology research.

Moderated Mediation Analysis

A moderated mediation analysis (Preacher, Rucker, & Hayes, 2007) was conducted to test our predictions that (1) the relation between instructors' perceptions of the importance of psychological research and their incorporation of this research into their elementary mathematics course is mediated by how much they access psychological research and (2) perceived comprehensibility acts as a moderator of these relations.

Table 5
Emphasis on Concepts for Which There Is Relevant Cognitive Psychology Research When Teaching About Arithmetic

	Percentage of instructors in each rating category					
	Not at all (%)	*Very little (%)*	*To some extent (%)*	*To a great extent (%)*	*Don't know/unsure (%)*	*Mean rating (SD)*[a]
Arithmetic development						
Typical developmental sequence of arithmetic strategies	0.55	7.14	45.60	46.70	0.00	3.38 (.64)
Factors that influence which strategies children use on a given problem (e.g., problem difficulty, confidence thresholds, task demands)	2.70	11.35	50.27	35.68	0.00	3.19 (.74)
Typical misconceptions and errors	1.62	7.03	51.35	40.00	0.00	3.30 (.67)
Cognitive processes that support the acquisition of more sophisticated strategies (e.g., encoding, analogical thinking, automaticity)	4.86	14.05	54.05	26.49	0.54	3.03 (.78)
Student arithmetic errors						
Students' mathematical knowledge and/or misconceptions (e.g., incomplete understanding of place value)	0.00	1.62	9.19	89.19	0.00	3.88 (.38)
General learning processes (e.g., working memory issues, lack of metacognition)	7.07	28.26	55.43	9.24	0.00	2.67 (.74)
Instructional principles that are related to error reduction (e.g., immediate versus delayed feedback, group versus mixed problem sets)	4.35	28.26	45.65	21.20	0.54	2.84 (.81)

[a]"Don't know or unsure" responses were set as missing for the calculation of means and standard deviations.

Comprehensibility did not act as a moderator of either the relation between perceived importance and accessing ($b = .19$, $s_b = .28$, $p > .05$) or the relation between accessing and incorporation ($b = -.02$, $s_b = .07$, $p > .05$).

Therefore, we conducted a simple mediation analysis, excluding comprehensibility, to test whether accessing mediates the relation between perceived importance and incorporation. As shown in Figure 1, perceived importance was significantly related to both incorporation, $b = 1.22$, $s_b = .14$, $p < .01$, and accessing, $b = .64$, $s_b = .17$, $p < .01$. The relation between perceived importance and incorporation was weaker, however, when both perceived importance and accessing were included in a single model predicting for incorporation: perceived importance, $b = 1.00$, $s_b = .13$, $p < .01$, and accessing, $b = .34$, $s_b = .06$, $p < .01$. We then used the bootstrapping method with bias-corrected confidence estimates to complete the analysis (Preacher & Hayes, 2008). The product of the coefficients (*ab*) for the indirect path from perceived importance to incorporation by way of accessing was significant (point estimate = 0.22; 95% bias-corrected confidence interval = 0.10 to 0.38).

Summary

In sum, the results indicated that mathematics methods instructors view cognitive studies of mathematics to be fairly important for mathematics education and that they emphasize topics prominent in psychology studies of mathematical thinking in their courses. However, they seldom access this research through primary or secondary sources. A mediation analysis indicated that mathematics methods instructors' perception of the importance of the research predicts their incorporation of it in their courses, and that this relation is partially mediated by their accessing of it.

DISCUSSION

Both researchers and educational policymakers have argued for increased application of cognitive science research for improving instructional practice. This study, however, is among the first to attempt to empirically measure the extent to which key findings from cognitive research are communicated to future teachers of elementary mathematics. The results suggest that cognitive research findings may not be consistently communicated to preservice teachers, at least in the context of courses discussing mathematics pedagogy, in which they could arguably be most meaningful. The variability in responses across instructors also suggests that the extent to which preservice teachers are learning about cognitive research is not standardized across teacher education programs.

The instructors of elementary mathematics methods courses surveyed in this study tended to be neutral about the importance of cognitive psychology research as a whole for

Table 6
Survey Items Used to Construct Variables for Each Construct of Research Utilization

Perceived importance

In your opinion, how important is each of the following in the preparation of preservice teachers who will teach elementary mathematics?

[*basic cognitive processes; basic learning processes; cognitive strategies and processes; common misconceptions; common developmental sequence; and predicting student performance from prior knowledge*]

Perceived comprehensibility

Please indicate your level of agreement with each of the following statements regarding various types of research. Please respond only as it relates to empirical research that has implications for elementary mathematics education.

Cognitive psychology journal articles are accessible (i.e., capable of being understood)

Developmental psychology journal articles are accessible (i.e., capable of being understood)

Accessing

How familiar are you with the work of each of the following authors?

[e.g., *David Geary, Martha Alibali, and Robert Siegler*]

How often do you read research findings from the following fields?

[*Learning Sciences or Educational Psychology and Cognitive/Developmental Psychology*]

How often do you read or reference articles or information (i.e., for your own research or professional development) from each of the following journals or databases?

[*Journal of Educational Psychology*, *Child Development*, *Cognition and Instruction, Cognitive Development*, *Journal of Applied Developmental Psychology, and Developmental Psychology*]

Please indicate your level of agreement with each of the following statements regarding empirical research from various fields. Please respond only as it relates to empirical research that has implications for elementary mathematics education.

I keep abreast of empirical research from the field of (cognitive/developmental psychology)

I read empirical research articles to access research from the field of (cognitive/developmental psychology)

I read practitioner-friendly articles to access summaries of research from the field of (cognitive/developmental psychology)

Incorporation into elementary mathematics methods course

How often did you ask students to do each of the following in the most recent math methods course that you taught?

[*analyze the skills and processes involved in mathematical tasks and identify student misconceptions and errors*]

To what extent did the most recent course that you taught emphasize each of the following?

[*the developmental sequence of mathematics concepts, and skills; cognitive strategies for mathematics problem solving, the mechanisms and processes underlying mathematics learning; and differences between typical and atypical learners in mathematics*]

When teaching about arithmetic in your most recent course that you taught, to what extent did you emphasize these concepts?

Imagine you presented the following student error to your class of preservice teachers:

$$\begin{array}{r} 17 \\ +18 \\ \hline 215 \end{array}$$

To what extent would you emphasize the concepts in a discussion of this student error?

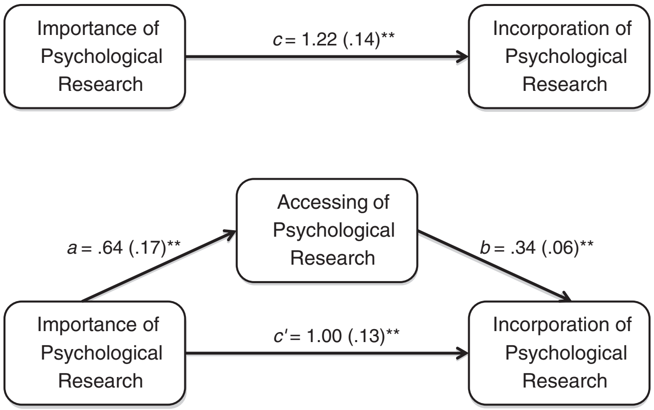

Fig. 1. Accessing as a mediator of the relation between perceived importance and incorporation.

mathematics education. Yet, as illustrated in Tables 2, 4, and 5, they generally recognized the importance of the kinds of topics prominent in psychology studies of mathematical thinking and emphasized these topics in their courses, particularly domain-specific topics. These data provide a better understanding of teacher educators' views about which topics may be most relevant to improving current mathematics instruction and most useful for elementary mathematics teachers to know. In that regard, the current results can point psychologists to topics for research or ways in which to frame their research findings for educators.

What might be done to facilitate a stronger connection between cognitive science and educational practice to increase the use of cognitive research in the training of elementary mathematics teachers across teacher education programs? The present results suggest that increasing mathematics education faculty's perception of the importance of cognitive psychology research for mathematics education may be crucial. We found that the greater mathematics education faculty perceives cognitive psychology research to be important to mathematics education, the more they emphasize psychological concepts in their courses. One particularly interesting result was the important role methods instructors' accessing of cognitive psychology research seems to play in their incorporation of it in their courses and the relatively insignificant role of perceived comprehensibility. As shown in Figure 1, accessing was strongly related to its incorporation in courses, whereas the perceived comprehensibility was unrelated to incorporation. Further, the relation between instructors' perceptions of the importance of cognitive research and their incorporation of it into their courses was partially mediated by the extent to which they accessed this research, but this relation was not moderated by perception of comprehensibility. Thus, instructors who felt that cognitive psychology research was important accessed this research more often regardless of whether or not they felt that the research was comprehensible. In addition, once teacher educators access relevant cognitive psychology research they are likely to incorporate key findings in their courses regardless of their perceptions of the comprehensibility of this work.

This result was surprising given the importance members of the educational community reportedly place on comprehensibility. It may be that comprehensibility was less of an issue in this study because of the focus on teacher educators as opposed to K-12 practitioners. Instructors in a university-based academic program are likely to be generally more comfortable reading research than K-12 practitioners. Further, they may have a more significant knowledge base that may make psychological research more comprehensible despite the differences in terminology, methodology, and presentation styles between the fields of education and psychology (Bransford et al., 1999; Honig & Coburn, 2008). On the other hand, data from the 2004 National Study of Postsecondary Faculty indicate that compared to faculty in other postsecondary program areas, faculty in teacher education programs are less likely to focus on research. The most recent data available indicates that <5% of full-time education faculty report research as their principal activity, more than 67% of full-time education faculty report teaching as their principal activity, and another 20% focus on administration (Forrest Cataldi, Fahimi, & Bradburn, 2005). Thus, the results may be a positive sign that differences in terminology and methodology between the fields of psychology and education, which may minimize comprehensibility of cognitive psychology research (Honig & Coburn, 2008), are not necessarily an impediment to its use by educators with less research experience (e.g., teachers and administrators). Rather, a potential barrier to accessing may simply be the venues in which cognitive psychology research is published.

Increasing Perceptions of Importance

On the basis of the current results, we recommend greater effort be made to raise awareness among teacher educators and teachers of the kinds of topics prominent in cognitive studies of mathematics thinking and learning. While numerous articles have been written by psychologists describing the implications of cognitive research for the instruction of mathematics (e.g., Newcombe et al., 2009), these are seldom published in periodicals likely to reach a large number of teacher educators, such as *American Educator*. Further, these articles tend to present discrete results as examples of the utility of cognitive research. An alternative approach is to identify and communicate general thematic categories of cognitive studies of mathematics, as was done in the introduction of this article and presented in Table 1. This approach could help educators recognize the broad range of work in cognitive psychology relevant to mathematics education and then to identify particular studies and researchers within those categories that would be most informative to their work.

A related recommendation is that psychology researchers make a greater effort to explain the possible implications and applications of their work, particularly for research that examines domain-general processes (e.g., encoding) because educators are more likely to perceive studies as valuable if they identify specific pedagogical strategies and techniques (Zeuli, 1994). One means of meeting this goal are resources such as the Institute of Education Sciences' *Practice Guides*. A second means may be greater collaboration between psychologists and educators—including teachers, educational leaders, teacher educators, and education researchers—while research is being conducted and prepared for publication to better specify the instructional implications of the findings.

In addition, we believe the present data suggest specific ways to make findings more relevant to mathematics educators by providing information about the topics perceived to be most relevant to mathematics instruction. For example, the majority of instructors surveyed indicated that they emphasized approaches for assessing mathematical thinking in their courses, but far fewer indicated that they emphasize the processes underlying mathematics learning (Table 4). To us, this suggests that if articles that focus on the cognitive processes underlying mathematics learning explicitly described how knowledge of these processes could be used to assess student learning, then the findings may seem more relevant and immediately applicable to educators.

Last, and perhaps most importantly, we recommend that further research about the value of including cognitive research in teacher training be conducted. The argument that cognitive science can improve instructional practice and student achievement has primarily been a theoretical and rhetorical one. The current results suggest that teacher educators may be skeptical about this argument.

Studies that have examined the influence of professional development that included student thinking and processes provide some empirical basis for the idea that knowledge of cognitive research improves teacher's mathematics instruction (Fennema et al., 1996; Saxe et al., 2001). However, our review of the literature found no studies that have examined the effect on preservice teachers' future instruction or their students' achievement.

Learning about cognitive research in the context of a teacher education program might potentially lead to the greatest impact on instruction—for instance, preservice teachers may have fewer preconceived notions about learning and more guidance from instructors about the application of the research to practice. On the other hand, it seems plausible that preservice education may be too early in teachers' careers for cognitive research to make the most impact on their practice—beginning teachers are often overwhelmed with learning the culture of schools and basic classroom management (Veenman, 1984). It may be possible that particular topics emerging from cognitive psychology are differentially useful for teachers with varying levels of experience and expertise. This information would be important for teacher educators who must determine which content is most important to include in their courses.

Increasing the Extent of Accessing

Instructors accessing of cognitive research findings mediated the influence of their perceived importance of this work on their incorporation of it in their courses. Thus, unless efforts are made to facilitate teacher educators' access to cognitive research, increasing their perceptions of its importance may be, at least partially, in vain.

It is not immediately apparent, however, how to achieve this goal. The factors that previous research suggested would be most likely to be related to accessing research—perceived importance and comprehensibility—predicted only a modest percentage of the variance in instructors' accessing. Thus, further research is needed to identify other potential factors, including individual and institutional level factors.

A factor not investigated in this study that might be important is instructors' confidence in the teaching resources available to them. Instructors who are more confident in the textbook or other trade books used for their course may feel less compelled to access additional research on their own. It also may be that instructors who perceive cognitive research to be important carefully select texts that include key principles and concepts from psychology and, therefore, do not perceive it necessary to access the research on their own. On the other hand, it may be that comfort in presenting the material in a textbook may afford instructors the time to supplement the text with additional research. Future research should include an analysis of the topics presented in books commonly used in elementary mathematics methods courses as well as instructors' satisfaction with and use of these texts.

This hypothesized reliance on textbooks is consistent with the current data indicating that while instructors do not frequently access research from the field of cognitive psychology they do incorporate some concepts from cognitive psychology, particularly mathematics specific topics, into their courses. If course textbooks are found to be an important avenue through which mathematics teacher educators learn about cognitive psychology research findings, then this would suggest that efforts to increase accessing might focus on disseminating key findings to key authors and publishers.

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NOTE

1 The results of all inferential analyses were identical when recoding "Don't know or unsure" as "Neither agree nor disagree" or "Never".

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